
MATHCOUNTS®

2016
■ Chapter Competition ■
Countdown Round
Problems 1–80

**This booklet contains problems to be used
in the Countdown Round.**

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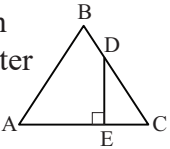
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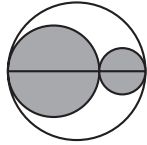
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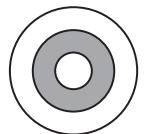
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1. _____ What is the least value of x^2 if $-20 \leq x \leq 16$?
2. _____ Two numbers have a sum of 1000 and a difference of 2. Exactly one of these numbers is prime. What is the composite number?
3. _____ (inches) A man who is exactly 6 feet tall has index fingers that are 3.6 inches long. If a 14-foot-tall statue of this man is built to scale, how many inches long should the statue's index fingers be? Express your answer as a decimal to the nearest tenth.
4. _____ (units) Equilateral triangle $\triangle ABC$ has side length 5 units. Segment DE is drawn perpendicular to \overline{AC} so that $EC = 2$ units. In units, what is the perimeter of quadrilateral $ABDE$? Express your answer in simplest radical form.
 
5. _____ The reciprocal of $x + 2$ equals $x - 2$ for what positive number x ? Express your answer in simplest radical form.
6. _____ Kathy's times in minutes and seconds for three 1-mile runs are 10:13, 9:51 and 9:38. Her average time is t seconds less than 10 minutes. What is the value of t ?
7. _____ (years old) Anita is three times Li's age but only one-third Owen's age. The sum of all three of their ages is 78 years. How old is Li?
8. _____ (rolls) Sydra repeatedly rolls two standard six-sided dice and records each resulting sum. What is the maximum number of rolls without repeating a sum?
9. _____ A regular octahedron is a polyhedron with eight congruent triangular faces. What is the sum of the number of edges and the number of vertices of a regular octahedron?
10. _____ (zeros) When the value of the expression $(0.1) \cdot (0.01) \cdot (0.001) \cdot (0.0001)$ is written in decimal form, how many zeros are between the decimal point and the digit 1?
11. _____ When two standard, six-sided dice are rolled, what is the probability that the two resulting numbers will be consecutive? Express your answer as a common fraction.
12. _____ What is the integer value of the expression $\sqrt{20} \cdot \sqrt{30} \cdot \sqrt{40} \cdot \sqrt{60}$?
13. _____ What is the least integer n for which $3^n > 2016$?
14. _____ (cm) If a triangle with integer side lengths has two sides measuring 18 cm and 25 cm, what is the shortest possible perimeter, in centimeters?
15. _____ What common fraction is equivalent to $1 + \frac{2}{3 + \frac{4}{5 + 6}}$?
16. _____ (times) Byron writes down all of the positive integers from 119 to 158. How many times does he write the digit 2?

17. _____ The figure shows three circles whose diameters are collinear. The two shaded circles are tangent to the largest circle and are also tangent to each other. The diameter of the larger shaded circle is twice that of the smaller shaded circle. What is the ratio of the combined area of the shaded circles to the area of the largest circle? Express your answer as a common fraction.

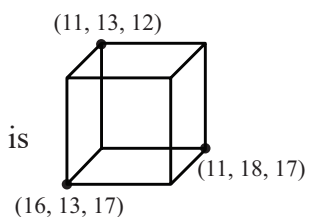


18. _____ If the mean of the numbers $3x + 10$, $7x + 10$ and $11x + 10$ is 73, what is the value of x ?
19. _____ (ways) In how many different ways can six identical balls be distributed into three jars labeled A, B and C, respectively, so that each jar contains at least one ball?
20. _____ (values) Three distinct positive integers less than 100 are added together. How many possible values are there for the sum?
21. _____ If $3x + 1 = 10$, what is the value of $6x + 1$?
22. _____ A committee is to be formed by randomly selecting three students from a group of two girls and three boys. What is the probability that at least one girl will be on the committee? Express your answer as a common fraction.
23. _____ (mm) A right triangle is inscribed in a circle. One angle of the triangle measures 30° and the length of the side opposite that angle is 14 mm. In millimeters, what is the circle's diameter?
24. _____ (mega bytes) On one computer, formatting any file takes 5 seconds and downloading a file takes $0.3m$ seconds, where m is the file size, in megabytes. On a different computer, formatting any file takes 3 seconds and downloading a file takes $0.5m$ seconds. For what file size, in megabytes, is the total time for formatting and then downloading a file the same for both computers?
25. _____ Each of the digits 1 through 8 is used exactly once to write two 4-digit integers. What is the greatest possible difference between the two integers?
26. _____ (meters) Zoey is training for a swim meet. During the first week of her training she swims 300 meters each day. Each week she increases the length of her daily swim by 100% of the previous week's daily length. How many meters will she swim each day during the fifth week of her training?
27. _____ The dartboard shown consists of three regions. The innermost circle has radius 1 inch, the middle circle has radius 4 inches and the outermost circle has radius 8 inches. What is the probability that a dart landing on the board is within the shaded region? Express your answer as common fraction.



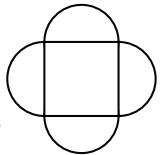
28. _____ What is the 24th term in the arithmetic sequence 4, 9, 14, 19, 24, ...?

29. _____ (ways) How many ways are there to make 14 cents using any combination of pennies, nickels and dimes?
30. _____ If $\frac{(2n)!}{n!} = 120$, what is the value of n ?
31. _____ (integers) How many integers between 1 and 500 are divisible by 3?
32. _____ Twenty-four balls numbered from 1 through 24 are colored alternately red, green, blue and yellow, in that order, then placed in a box. If a ball is randomly drawn from the box, what is the probability that it is blue and has a prime number? Express your answer as a common fraction.
33. _____ (inches) A rectangular prism has length 20 inches and width 5 inches. If the surface area of the prism is 1100 in^2 , what is the height of the prism, in inches?
34. _____ When the digits of 4192 are rearranged to form a different 4-digit positive integer, what is the least possible absolute difference between the new integer and 4192?
35. _____ If $x > 1$ and $(x^6)^3 \cdot (x^k)^3 \cdot (x^{24})^3 = x^{300}$, what is the value of k ?
36. _____ (ft^3) How many cubic feet are in the volume of a rectangular box with dimensions 9 inches by 2 feet by 2 yards?
37. _____ (sums) Two fair six-sided dice each have faces numbered 1, 1, 2, 3, 5 and 8. If both dice are rolled, how many distinct sums are possible?
38. _____ (dollars) During a ten-day period, Omar spends an average of \$18 per day. If he spends at least \$10 each day, what is the greatest number of dollars that he could spend on any one day?
39. _____ What is the sum of the distinct prime factors of 9999?
40. _____ (units^3) The coordinates of three vertices of a cube are shown. What is the volume of the cube, in cubic units?
41. _____ What is the value of $\frac{1}{2} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8}$? Express your answer as a common fraction.
42. _____ (mi/h) The one-way distance from Sisters, Oregon to the top of the McKenzie Pass is 15 miles. Ember took 2.5 hours to ride her bike from Sisters to the top of the pass, and her return trip took 30 minutes. What was Ember's average speed, in miles per hour, for the entire trip?
43. _____ Willow bakes a small cake and eats $\frac{1}{6}$ of it. Caroline then eats $\frac{1}{4}$ of what is left. What fraction of the cake remains after Caroline eats her share? Express your answer as a common fraction.



44. _____ (inches) A particular circle has a radius greater than 0. The number of square inches in the area of the circle is 6 times the number of inches in its circumference. What is the circle's radius, in inches?
45. _____ If $a^2 + 2ab + b^2 = 16$ and $c^2 + 2cd + d^2 = 36$, what is the least possible value of the sum $a + b + c + d$?
46. _____ What is the 2016th positive even integer?
47. _____ (miles) A taxi charges \$2.50 for the first $\frac{1}{3}$ mile and \$0.45 for each $\frac{1}{6}$ mile thereafter. How many miles would the shortest ride costing \$7.90 be? Express your answer as a mixed number.
48. _____ The height of a cone is three times its base radius. A smaller cone is created by slicing parallel to the base so that the height of the smaller cone equals the base radius of the larger cone. What is the ratio of the volume of the smaller cone to the volume of the larger cone? Express your answer as a common fraction.
49. _____ (yards/minute) If a snake travels at a constant rate of 1 foot per second, how many yards per minute does it travel?
50. _____ (meters) Bjorn walks 40 laps around a circle of diameter 70 meters. Anna walks 30 laps around a square of side length 80 meters. Carol walks 50 laps around an equilateral triangle of side length 60 meters. In meters, what is the greatest distance that any one person walks?
51. _____ (minutes) It takes Leo 54 minutes to mow $\frac{3}{4}$ of his lawn. How many minutes does it take him to mow his entire lawn?
52. _____ If $x \Delta y = x + y - |x - y|$, what is the value of $(3 \Delta 4) - (2 \Delta 1)$?
53. _____ (yards) The length of two pieces of string laid end to end is 60 yards. The absolute difference between the lengths of the two pieces of string is 30 yards. How many yards long is the longer piece of string?
54. _____ (integers) How many 3-digit positive, odd integers are multiples of 5?
55. _____ (minutes) On Mondays, Wednesdays and Fridays Kole swims for 45 minutes. On Tuesdays and Thursdays he swims for 60 minutes. What is the mean number of minutes that Kole swims daily Monday through Friday?
56. _____ (miles) Abdul lives 4 miles west and 2 miles north of Central Middle School. Jamal lives 8 miles east and 7 miles north of the same school. What is the shortest distance, in miles, between Abdul's house and Jamal's house?



57. _____ (inches) Kent walks 15,000 feet per hour. How many inches does he walk each second?
58. _____ US dollar bills are printed in sheets of 32 bills with 4 columns and 8 rows per sheet. What is the probability that a given dollar bill was along an edge of its original sheet? Express your answer as a common fraction.
59. _____ The figure shows a square of side length 2 inches surrounded by four semicircles, each with a side of the square as its diameter. The total area of the figure can be written in simplest radical form, in terms of π , as $a + b\pi$ in². What is the value of $a + b$?
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60. _____ Using each of the digits 1, 5, 6 and 9 exactly once, what decimal number can be written that is closest in value to 60? Express your answer as a decimal to the nearest hundredth.
61. _____ The average of Len's grades on seven homework assignments is 83. If Len earned a grade of 79 on each of the first five assignments, what is the average of his grades on the last two assignments?
62. _____ (students) Each of the 26 eighth-grade students at Monarch Middle School is in orchestra, algebra or both. Of these students, 20 are in orchestra and 16 are in algebra. How many students are in both orchestra and algebra?
63. _____ (children) A day care facility has a rectangular play area that is 30 feet by 35 feet. Guidelines require a minimum of 48 ft² of usable activity area per child. What is the maximum number of children that can occupy this play area?
64. _____ Ky has a bag with 4 pieces of red and 2 pieces of green candy. He has another bag with 6 pieces of red and 4 pieces of green candy. If Ky randomly takes one piece of candy from each bag, what is the probability that both pieces of candy will be red? Express your answer as a common fraction.
65. _____ (ml) A mixture of 1% hydrochloric acid measures 24 ml. How many milliliters of 5% hydrochloric acid must be added to change the mixture to 4% hydrochloric acid?
66. _____ (inches) A photo that measures $4\frac{1}{2}$ inches long by $6\frac{1}{2}$ inches wide is re-sized to make a similar photo that is 6 inches long. How many inches wide is the re-sized photo? Express your answer as a mixed number.
67. _____ (°F) The average temperature in Chicago on January 3rd was -10 °F. The average temperature increased by 3 degrees every 7 days. In degrees Fahrenheit, what was the average temperature on January 24th?

68. _____ (°F) The number of times N that a cricket chirps per minute is given by the formula $T = 40 + \frac{N}{4}$, where T is the temperature in degrees Fahrenheit. What is the temperature in degrees Fahrenheit when a cricket chirps 36 times in 15 seconds?
69. _____ (units²) In square units, what is the greatest possible area of a triangle with perimeter 36 units? Express your answer in simplest radical form.
70. _____ What is the value of $12(p + 9) - 12(p - 3)$?
71. _____ Ten numbers have a mean of 32. Eight of the ten have a mean of 30 and the remaining two numbers differ by 4. What is the greater of these two numbers?
72. _____ (matches) The annual chess club tournament begins with 64 students in the first round. In each round, students are paired up for chess matches. The winner of each match advances to the next round and the loser is out of the tournament. How many matches will be played in the second round?
73. _____ (candies) Hannah has a bag with 8 red, 6 blue and 2 green candies. What is the least number of candies she must take from the bag, without looking, to ensure that she has at least 2 of the same color?
74. _____ (questions) Ms. Clark's semester exam consists of true-false questions, worth 3 points each, and essay questions, worth 11 points each. There are 20 questions worth a total of 100 points. How many of the exam questions are true-false questions?
75. _____ (cups) Louis has 12 sled dogs and 6 puppies. Each sled dog eats $2\frac{1}{2}$ cups of dog food twice a day and each puppy eats $\frac{2}{3}$ cup of dog food twice a day. How many cups of dog food does Louis need each day to feed the dogs and puppies?
76. _____ (percent) Last year, Martha invested \$12,000. She had \$4000 that earned 6% annual interest and \$6000 that earned 5% interest. If she earned \$600 in total interest, what annual percent interest did the rest of her money earn?
77. _____ The sum of the digits of a positive 2-digit number is 7. Reversing the digits creates a new 2-digit number that is 9 more than the original number. What is the original number?
78. _____ (inches) Fernando builds a cube out of $\frac{5}{8}$ -inch sugar cubes and another cube out of 1-inch sugar cubes. If the two constructed cubes are the same size, what is the least possible edge length, in inches, of the cubes he builds?
79. _____ (dollars) The value of Bill's estate doubles at the end of every year. Its value at the end of the third year is \$8000. In dollars, what is its value at the end of the sixth year?
80. _____ (♥) If $3\clubsuit = 1\diamond$, $2\heartsuit = 1\clubsuit$ and $1\diamond = 2\spadesuit$, how many ♥ are equivalent to $1\spadesuit$?

