MATHCOUNTS®

2020
■ Chapter Competition ■
Team Round
Problems 1–10

School _____

Team Members _____, Captain

_____

_____

_____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

1. $\_\_\_\_\_$

Jack goes on a road trip across the Southwest, during which he buys fuel in several cities. The table shows the amounts and prices of the fuel Jack bought in each city. What is the greatest amount Jack spent on fuel in any of these cities?

Road Trip Fuel Purchases

City	Gallons	Price (per gallon)
Las Cruces, NM	10.44	2.47
Flagstaff, AZ	10.39	2.65

2. $\_\_\_\_\_$

23				
			26	

For the 5 × 5 array shown, Chad fills in each empty cell with a positive integer so that any two cells that are vertically or horizontally adjacent contain numbers that differ by exactly 1. What is the greatest number that can appear in any of these cells?

3. $\_\_\_\_\_$ times

Marko's heart beats 72 times per minute during normal activity. It beats 65 times per minute during sleep, and 118 times per minute when Marko is exercising. During a 24-hour period, Marko slept from 10:00 p.m. until 5:00 a.m. and then exercised from 5:30 a.m. to 7:30 a.m. For the remaining time he had normal activity. During that 24-hour period, how many times did Marko's heart beat?

4. $\_\_\_\_\_$ years

Kepler's third law of planetary motion states that the square of the amount of time it takes for a planet to orbit the Sun is proportional to the cube of the planet's greatest distance from the Sun. Based on this and the data for Mars, Jupiter, Uranus and Neptune included in the table shown, how many Earth years does it take for Neptune to orbit the Sun? Express your answer to the nearest whole number.

Sun Distances and Orbit Times

Planet	Greatest Distance (kilometers to Sun)	Orbit Time (Earth years)
Mars	2.28×10^8	1.882
Jupiter	7.78×10^8	11.86
Uranus	2.87×10^9	84.01
Neptune	4.50×10^9	?

5. $\_\_\_\_\_$

An ordered triple (a, b, c) is randomly chosen from the set of all ordered triples for which a, b and c are nonnegative integers that satisfy $a + b + c = 22$. What is the probability that $a < b < c$? Express your answer as a common fraction.

6. _____ sequences

Lincoln stands at vertex A of hexagon ABCDEF and rolls a die three times. For each roll, if the number rolled is even, he moves clockwise that number of vertices. If the number rolled is odd, he moves counter-clockwise that number of vertices. How many different sequences of three rolls will result in Lincoln ending at vertex A?

7. _____

An abundant number is a positive integer n for which the sum of the positive integer factors of n is greater than $2n$. What is the sum of all the abundant numbers that are less than 60?

8. _____ lengths

Suppose 7 points lie in a plane, and a line segment is drawn connecting each pair of points, forming a total of 21 line segments. What is the minimum number of distinct lengths among the 21 line segments?

9. _____

A standard 52-card deck of playing cards has thirteen ranks in each of four suits. If five cards are randomly selected from the deck, without replacement, what is the probability that they are all the same suit? Express your answer as a decimal to the nearest thousandth.

10. _____ units^2

Let R be the set of all points (x, y) satisfying $x^2 + y^2 \leq 100$. What is the least possible area of a right triangle with three integer side lengths that cannot fit inside R?