MATHCOUNTS®

2023 STATE COMPETITION
Team Round Problems 1−10

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3
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School _____

Chapter _____

Team
Members _____ , Captain

_____

_____

_____

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DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

03-S23TEA

1. _____ If $\blacksquare + \blacklozenge = 18$, $\left(\!\! + \left(\!\! + \blacksquare = 13\right.\right.$, and $\left(\!\! + \left(\!\! + \left(\!\! = 1.5\right.\right.\right.$, what is the value of $\blacklozenge$?

2. _____ 7th graders   Archie surveyed all of the students at Riverdale Middle School to determine what eye color is most common. On the survey, all students answered one question asking them to choose their eye color between blue, brown, and green eyes (with no other choices). Unfortunately, he lost part of his data, and only has the incomplete chart shown. How many 7th graders with brown eyes are there at Riverdale Middle School?

	6th	7th	8th	Total
Blue	38	?	28	97
Brown	?	?	?	339
Green	20	?	15	59
Total	157	176	162	495

3. _____ A six digit number is written as 839A6B where A and B are two unknown digits. If the number is divisible by 22 and 4, what is the greatest possible value of $|A - B|$?

4. _____ %   Four fully inflated spherical beach balls are packed snugly in a rectangular prism box as shown. What percent of the volume of the box is occupied by the beach balls? Express your answer to the nearest whole percent.
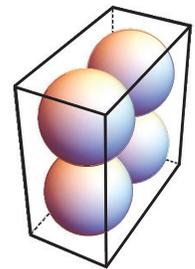
5. _____ units²   In the figure shown, the circle is inscribed into a quarter circle. If the area of the circle is π, then what is the area of the shaded region? Express your answer as a decimal to the nearest tenth.
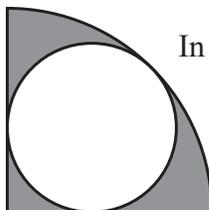
6. _____ ordered pairs   How many ordered pairs of positive integers (x, y) satisfy $x \leq 2y \leq 20$ and $y \leq 2x \leq 20$?

7. _____   Three flexible rings are arranged as shown on the left. At each of the six points where two rings cross, one of the rings is randomly chosen to pass over the other. One such arrangement of the three rings is shown on the right. What is the probability that all three rings can be pulled apart and separated? Express your answer as a common fraction.
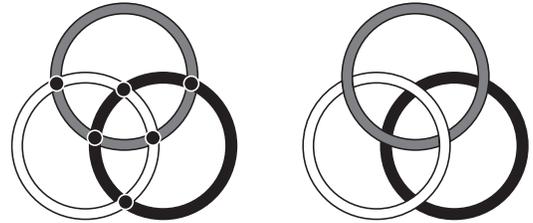
8. _____ config-urations   Seven distinct lightbulbs are arranged in a circle. Each bulb can be either on or off. In order to properly light up the room, in each group of three adjacent light bulbs, at least one must be turned on. How many such configurations are there?

9. _____   A positive integer is called triskaidekaphilic if it is equal to 13 times a number resulting by removing one digit from it and writing the remaining digits in order. For example, 195 and 325 are both triskaidekaphilic because 195 = 13 × 15 and 325 = 13 × 25. What is the sum of all the triskaidekaphilic numbers between 1000 and 9999 inclusive?

10. _____ cm   Right triangle ABC has right angle at A and AB = 2 cm. Equilateral triangles ABD, ACE, and BCF are constructed outward along the sides of ABC, as shown. If triangle DEF has area $23\sqrt{3}$ cm², what is the length of the hypotenuse of triangle ABC? Express your answer in simplest radical form.